CHAPTER

# 4

# CONGRUENCE OF LINE SEGMENTS, ANGLES, AND TRIANGLES

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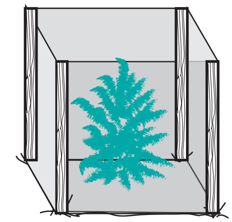
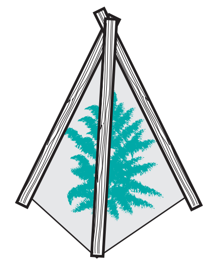
One of the common notions stated by Euclid was the following: "Things which coincide with one another are equal to one another." Euclid used this common notion to prove the congruence of triangles. For example, Euclid's Proposition 4 states, "If two triangles have the two sides equal to two sides respectively, and have the angles contained by the equal straight lines equal, they will also have the base equal to the base, the triangle will be equal to the triangle, and the remaining angles will be equal to the remaining angles respectively, namely those which the equal sides subtend." In other words, Euclid showed that the equal sides and angle of the first triangle can be made to coincide with the sides and angle of the second triangle so that the two triangles will coincide. We will expand on Euclid's approach in our development of congruent triangles presented in this chapter.

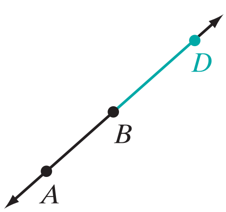
## 4-1 POSTULATES OF LINES, LINE SEGMENTS, AND ANGLES

Melissa planted a new azalea bush in the fall and wants to protect it from the cold and snow this winter. She drove four parallel stakes into the ground around the bush and covered the structure with burlap fabric. During the first winter storm, this protective barrier was pushed out of shape. Her neighbor suggested that she make a tripod of three stakes fastened together at the top, forming three triangles. Melissa found that this arrangement was able to stand up to the storms. Why was this change an improvement? What geometric figure occurs most frequently in weight-bearing structures? In this chapter we will study the properties of triangles to discover why triangles keep their shape.

Recall that a line, $\overleftrightarrow{AB}$, is an infinite set of points that extends endlessly in both directions, but a line segment, $\overline{AB}$, is a part of $\overleftrightarrow{AB}$ and has a finite length. We can choose some point of $\overleftrightarrow{AB}$ that is not a point of $\overline{AB}$ to form a line segment of any length. When we do this, we say that we are extending the line segment.

**Postulate 4.1** A line segment can be extended to any length in either direction.




When we choose point $D$ on $\overleftrightarrow{AB}$ so that $B$ is the midpoint of $\overline{AD}$, we say that we have extended $\overline{AB}$ but $\overline{AD}$ is *not* the original segment, $\overline{AB}$. In this case, we have chosen $D$ so that $AB = BD$ and $AD = 2AB$.

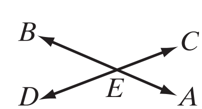
We will also accept the following postulates:

**Postulate 4.2** Through two given points, one and only one line can be drawn.
*Two points determine a line.*

Through given points $C$ and $D$, one and only one line can be drawn.

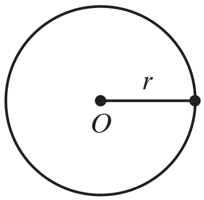

**Postulate 4.3** Two lines cannot intersect in more than one point.

$\overleftrightarrow{AEB}$ and $\overleftrightarrow{CED}$ intersect at $E$ and cannot intersect at any other point.



**Postulate 4.4**

> One and only one circle can be drawn with any given point as center and the length of any given line segment as a radius.



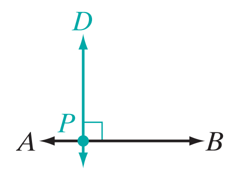

Only one circle can be drawn that has point $O$ as its center and a radius equal in length to segment $r$.

We make use of this postulate in constructions when we use a compass to locate points at a given distance from a given point.

**Postulate 4.5**

> At a given point on a given line, one and only one perpendicular can be drawn to the line.

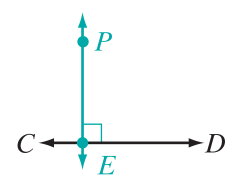
At point $P$ on $\overleftrightarrow{APB}$, exactly one line, $\overleftrightarrow{PD}$, can be drawn perpendicular to $\overleftrightarrow{APB}$ and no other line through $P$ is perpendicular to $\overleftrightarrow{APB}$.




**Postulate 4.6**

> From a given point not on a given line, one and only one perpendicular can be drawn to the line.

From point $P$ not on $\overleftrightarrow{CD}$, exactly one line, $\overleftrightarrow{PE}$, can be drawn perpendicular to $\overleftrightarrow{CD}$ and no other line from $P$ is perpendicular to $\overleftrightarrow{CD}$.




**Postulate 4.7**

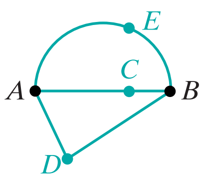
> For any two distinct points, there is only one positive real number that is the length of the line segment joining the two points.


For the distinct points $A$ and $B$, there is only one positive real number, represented by $AB$, which is the length of $\overline{AB}$.

Since $AB$ is also called the distance from $A$ to $B$, we refer to Postulate 4.7 as the **distance postulate**.

**Postulate 4.8**

> The shortest distance between two points is the length of the line segment joining these two points.




The figure shows three paths that can be taken in going from $A$ to $B$.

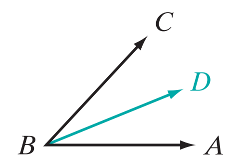
The length of $\overline{AB}$ (the path through $C$, a point collinear with $A$ and $B$) is less than the length of the path through $D$ or the path through $E$. The measure of the shortest path from $A$ to $B$ is the distance $AB$.

**Postulate 4.9** A line segment has one and only one midpoint.

$\overline{AB}$ has a midpoint, point $M$, and no other point is a midpoint of $\overline{AB}$.

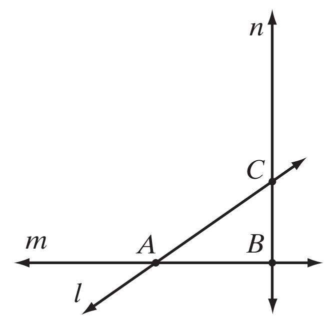

**Postulate 4.10** An angle has one and only one bisector.

Angle $ABC$ has one bisector, $\overrightarrow{BD}$, and no other ray bisects $\angle ABC$.




## EXAMPLE 1

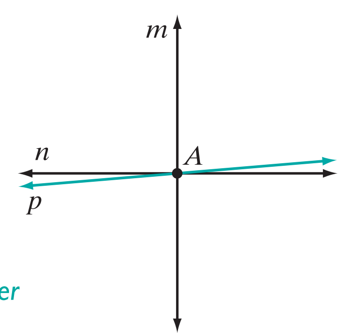
Use the figure to answer the following questions:




| | *Answers* |
|---|---|
| **a.** What is the intersection of $m$ and $n$? | point $B$ |
| **b.** Do points $A$ and $B$ determine line $m$, $n$, or $l$? | line $m$ |

## EXAMPLE 2

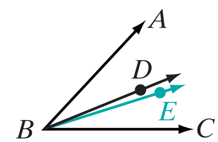
Lines $p$ and $n$ are two distinct lines that intersect line $m$ at $A$. If line $n$ is perpendicular to line $m$, can line $p$ be perpendicular to line $m$? Explain.




***Solution*** No. Only one perpendicular can be drawn to a line at a given point on the line. Since line $n$ is perpendicular to $m$ and lines $n$ and $p$ are distinct, line $p$ cannot be perpendicular to $m$. *Answer*

**EXAMPLE 3**

If $\overrightarrow{BD}$ bisects $\angle ABC$ and point $E$ is not a point on $\overrightarrow{BD}$, can $\overrightarrow{BE}$ be the bisector of $\angle ABC$?




***Solution*** No. An angle has one and only bisector. Since point $E$ is not a point on $\overrightarrow{BD}$, $\overrightarrow{BD}$ is not the same ray as $\overrightarrow{BE}$. Therefore, $\overrightarrow{BE}$ cannot be the bisector of $\angle ABC$. *Answer*

## Conditional Statements as They Relate to Proof

To prove a statement in geometry, we start with what is known to be true and use definitions, postulates, and previously proven theorems to arrive at the truth of what is to be proved. As we have seen in the text so far, the information that is known to be true is often stated as *given* and what is to be proved as *prove*. When the information needed for a proof is presented in a conditional statement, we use the information in the hypothesis to form a *given* statement, and the information in the conclusion to form a *prove* statement.

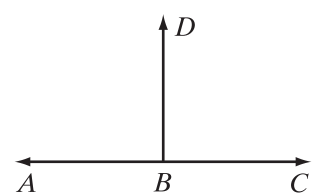
### Numerical and Algebraic Applications

**EXAMPLE 4**

Rewrite the conditional statement in the *given* and *prove* format:

If a ray bisects a straight angle, it is perpendicular to the line determined by the straight angle.

***Solution*** Draw and label a diagram.




Use the hypothesis, "a ray bisects a straight angle," as the *given*. Name a straight angle using the three letters from the diagram and state in the *given* that this angle is a straight angle. Name the ray that bisects the angle, using the vertex of the angle as the endpoint of the ray that is the bisector. State in the *given* that the ray bisects the angle:

*Given:* $\angle ABC$ is a straight angle and $\overrightarrow{BD}$ bisects $\angle ABC$.

Use the conclusion, "if (the bisector) is perpendicular to the line determined by the straight angle," to write the *prove*. We have called the bisector $\overrightarrow{BD}$, and the line determined by the straight angle is $\overleftrightarrow{ABC}$.

*Prove:* $\overrightarrow{BD} \perp \overleftrightarrow{AC}$

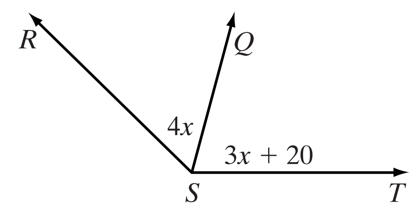
*Answer* *Given:* $\angle ABC$ is a straight angle and $\overrightarrow{BD}$ bisects $\angle ABC$.

*Prove:* $\overrightarrow{BD} \perp \overleftrightarrow{AC}$

In geometry, we are interested in proving that statements are true. These true statements can then be used to help solve numerical and algebraic problems, as in Example 5.

**EXAMPLE 5**

$\overrightarrow{SQ}$ bisects $\angle RST$, $m\angle RSQ = 4x$, and $m\angle QST = 3x + 20$. Find the measures of $\angle RSQ$ and $\angle QST$.




***Solution*** The bisector of an angle separates the angle into two congruent angles. Therefore, $\angle RSQ \cong \angle QST$. Then since congruent angles have equal measures, we may write an equation that states that $m\angle RSQ = m\angle QST$.

$$4x = 3x + 20$$
$$x = 20$$

$$m\angle RSQ = 4x = 4(20) = 80$$

$$m\angle QST = 3x + 20 = 3(20) + 20 = 60 + 20 = 80$$

*Answer* $m\angle RSQ = m\angle QST = 80$

## Exercises

### Writing About Mathematics

**1.** If two distinct lines $\overleftrightarrow{AEB}$ and $\overleftrightarrow{CED}$ intersect at a point $F$, what must be true about points $E$ and $F$? Use a postulate to justify your answer.

**2.** If $LM = 10$, can $\overline{LM}$ be extended so that $LM = 15$? Explain why or why not.

## Developing Skills

In 3–12, in each case: **a.** Rewrite the conditional statement in the *Given/Prove* format. **b.** Write a formal proof.

**3.** If $AB = AD$ and $DC = AD$, then $AB = DC$.

**4.** If $\overline{AD} \cong \overline{CD}$ and $\overline{BD} \cong \overline{CD}$, then $\overline{AD} \cong \overline{BD}$.

**5.** If $m\angle 1 + m\angle 2 = 90$ and $m\angle A = m\angle 2$, then $m\angle 1 + m\angle A = 90$.

**6.** If $m\angle A = m\angle B$, $m\angle 1 = m\angle B$, and $m\angle 2 = m\angle A$, then $m\angle 1 = m\angle 2$.

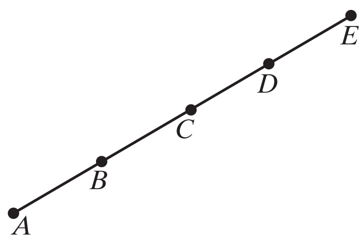
**7.** If $\overline{AB} \cong \overline{CD}$, and $\overline{EF} \cong \overline{CD}$, then $\overline{AB} \cong \overline{EF}$.

**8.** If $2EF = DB$ and $GH = \frac{1}{2}DB$, then $EF = GH$.

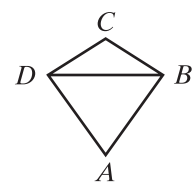
**9.** If $CE = CF$, $CD = 2CE$, and $CB = 2CF$, then $CD = CB$.

**10.** If $RT = RS$, $RD = \frac{1}{2}RT$, and $RE = \frac{1}{2}RS$, then $RD = RE$.

**11.** If $AD = BE$ and $BC = CD$, then $AC = CE$.




**12.** If $\angle CDB \cong \angle CBD$ and $\angle ADB \cong \angle ABD$, then $\angle CDA \cong \angle CBA$.

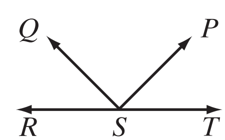



In 13–15, *Given:* $\overleftrightarrow{RST}$, $\overrightarrow{SQ}$, and $\overrightarrow{SP}$.

**13.** If $\angle QSR \cong \angle PST$ and $m\angle QSP = 96$, find $m\angle QSR$.

**14.** If $m\angle QSR = 40$ and $\overrightarrow{SQ} \perp \overrightarrow{SP}$, find $m\angle PST$.

**15.** If $m\angle PSQ$ is twice $m\angle QSR$ and $\overrightarrow{SQ} \perp \overrightarrow{SP}$, find $m\angle PST$.




## Applying Skills

In 16–19, use the given conditional to **a.** draw a diagram with geometry software or pencil and paper, **b.** write a *given* and a *prove*, **c.** write a proof.

**16.** If a triangle is equilateral, then the measures of the sides are equal.

**17.** If $E$ and $F$ are distinct points and two lines intersect at $E$, then they do not intersect at $F$.

**18.** If a line through a vertex of a triangle is perpendicular to the opposite side, then it separates the triangle into two right triangles.

**19.** If two points on a circle are the endpoints of a line segment, then the length of the line segment is less than the length of the portion of the circle (the arc) with the same endpoints.

**20.** Points $F$ and $G$ are both on line $l$ and on line $m$. If $F$ and $G$ are distinct points, do $l$ and $m$ name the same line? Justify your answer.

## 4-2 USING POSTULATES AND DEFINITIONS IN PROOFS

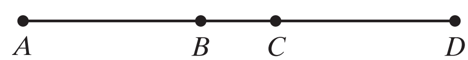
A *theorem* was defined in Chapter 3 as a statement proved by deductive reasoning. We use the laws of logic to combine definitions and postulates to prove a theorem. A carefully drawn figure is helpful in deciding upon the steps to use in the proof. However, recall from Chapter 1 that we cannot use statements that appear to be true in the figure drawn for a particular problem. For example, we may not assume that two line segments are congruent or are perpendicular because they appear to be so in the figure.

On the other hand, unless otherwise stated, we will assume that lines that appear to be straight lines in a figure actually are straight lines and that points that appear to be on a given line actually are on that line in the order shown.

### EXAMPLE 1

*Given:* $\overleftrightarrow{ABCD}$ with $\overline{AB} \cong \overline{CD}$

*Prove:* $\overline{AC} \cong \overline{BD}$

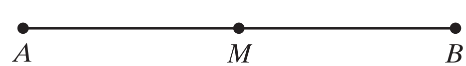



**Proof**

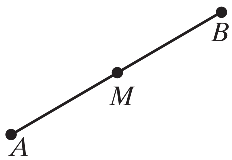
| Statements | Reasons |
|---|---|
| **1.** $A$, $B$, $C$, and $D$ are collinear with $B$ between $A$ and $C$ and $C$ between $B$ and $D$. | **1.** Given. |
| **2.** $\overline{AB} + \overline{BC} = \overline{AC}$<br>$\overline{BC} + \overline{CD} = \overline{BD}$ | **2.** Partition postulate. |
| **3.** $\overline{AB} \cong \overline{CD}$ | **3.** Given. |
| **4.** $\overline{BC} \cong \overline{BC}$ | **4.** Reflexive property. |
| **5.** $\overline{AB} + \overline{BC} \cong \overline{CD} + \overline{BC}$ | **5.** Addition postulate. |
| **6.** $\overline{AC} \cong \overline{BD}$ | **6.** Substitution postulate. |

### EXAMPLE 2

*Given:* $M$ is the midpoint of $\overline{AB}$.




*Prove:* $AM = \frac{1}{2}AB$ and $MB = \frac{1}{2}AB$



**Proof**

| Statements | Reasons |
|---|---|
| **1.** $M$ is the midpoint of $\overline{AB}$. | **1.** Given. |
| **2.** $\overline{AM} \cong \overline{MB}$ | **2.** Definition of midpoint. |
| **3.** $AM = MB$ | **3.** Definition of congruent segments. |
| **4.** $AM + MB = AB$ | **4.** Partition postulate. |
| **5.** $AM + AM = AB$ or $2AM = AB$ | **5.** Substitution postulate. |
| **6.** $AM = \frac{1}{2}AB$ | **6.** Halves of equal quantities are equal. |
| **7.** $MB + MB = AB$ or $2MB = AB$ | **7.** Substitution postulate. |
| **8.** $MB = \frac{1}{2}AB$ | **8.** Halves of equal quantities are equal. |

**Note:** We used definitions and postulates to prove statements about length. We could *not* use information from the diagram that appears to be true.

## Exercises

### Writing About Mathematics

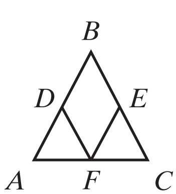
**1.** Explain the difference between the symbols $\overline{ABC}$ and $\overline{ACB}$. Could both of these describe segments of the same line?

**2.** Two lines, $\overleftrightarrow{ABC}$ and $\overleftrightarrow{DBE}$, intersect and m$\angle ABD$ is 90. Are the measures of $\angle DBC$, $\angle CBE$, and $\angle EBA$ also 90? Justify your answer.

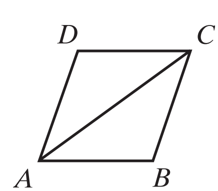
### Developing Skills

In 3–12, in each case: **a.** Rewrite the conditional statement in the *Given/Prove* format. **b.** Write a proof that demonstrates that the conclusion is valid.

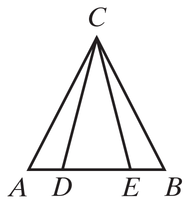
**3.** If $\overline{AB} \cong \overline{CB}$, $\overline{FD}$ bisects $\overline{AB}$, and $\overline{FE}$ bisects $\overline{CB}$, then $\overline{AD} \cong \overline{CE}$.



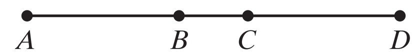

**4.** If $\overrightarrow{CA}$ bisects $\angle DCB$, $\overrightarrow{AC}$ bisects $\angle DAB$, and $\angle DCB \cong \angle DAB$, then $\angle CAB \cong \angle DCA$.



**5.** If $\overline{AD} \cong \overline{BE}$, then $\overline{AE} \cong \overline{BD}$.

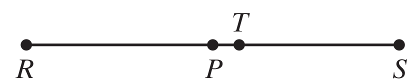



**6.** If $\overline{ABCD}$ is a segment and $AB = CD$, then $AC = BD$.

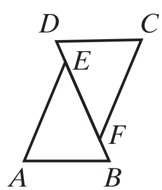



**7.** If $\overline{ABCD}$ is a segment, $B$ is the midpoint of $\overline{AC}$, and $C$ is the midpoint of $\overline{BD}$, then $AB = BC = CD$.

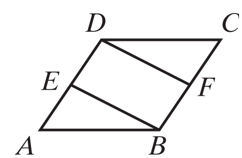

**8.** If $P$ and $T$ are distinct points and $P$ is the midpoint of $\overline{RS}$, then $T$ is not the midpoint of $\overline{RS}$.



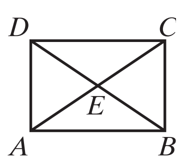

**9.** If $\overline{DF} \cong \overline{BE}$, then $\overline{DE} \cong \overline{BF}$.



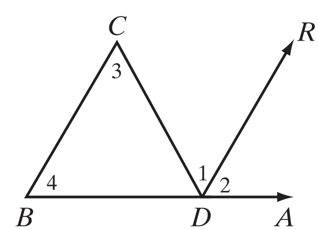

**10.** If $\overline{AD} \cong \overline{BC}$, $E$ is the midpoint of $\overline{AD}$, and $F$ is the midpoint of $\overline{BC}$, then $\overline{AE} \cong \overline{FC}$.




**11.** If $\overline{AC} \cong \overline{DB}$ and $\overline{AC}$ and $\overline{DB}$ bisect each other at $E$, then $\overline{AE} \cong \overline{EB}$.




**12.** If $\overrightarrow{DR}$ bisects $\angle CDA$, $\angle 3 \cong \angle 1$, and $\angle 4 \cong \angle 2$, then $\angle 3 \cong \angle 4$.




## Applying Skills

**13.** The rays $\overrightarrow{DF}$ and $\overrightarrow{DG}$ separate $\angle CDE$ into three congruent angles, $\angle CDF$, $\angle FDG$, and $\angle GDE$. If $m\angle CDF = 7a + 10$ and $m\angle GDE = 10a - 2$, find:

**a.** $m\angle CDG$ **b.** $m\angle FDE$ **c.** $m\angle CDE$

**d.** Is $\angle CDE$ acute, right, or obtuse?

**14.** Segment $\overline{BD}$ is a bisector of $\overline{ABC}$ and is perpendicular to $\overline{ABC}$. $AB = 2x - 30$ and $BC = x - 10$.

**a.** Draw a diagram that shows $\overline{ABC}$ and $\overline{BD}$.

**b.** Find $AB$ and $BC$.

**c.** Find the distance from $A$ to $\overline{BD}$. Justify your answer.

**15.** Two line segments, $\overline{RS}$ and $\overline{LM}$, bisect each other and are perpendicular to each other at $N$, and $RN = LN$. $RS = 3x + 9$, and $LM = 5x - 17$.

**a.** Draw a diagram that shows $\overline{RS}$ and $\overline{LM}$.

**b.** Write the *given* using the information in the first sentence.

**c.** Prove that $RS = LM$.

**d.** Find $RS$ and $LM$.

**e.** Find the distance from $L$ to $\overline{RS}$. Justify your answer.

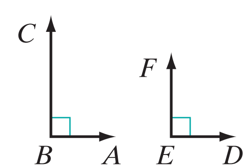
## 4-3 PROVING THEOREMS ABOUT ANGLES

In this section we will use definitions and postulates to prove some simple theorems about angles. Once a theorem is proved, we can use it as a *reason* in later proofs. Like the postulates, we will number the theorems for easy reference.

**Theorem 4.1** If two angles are right angles, then they are congruent.

*Given* $\angle ABC$ and $\angle DEF$ are right angles.

*Prove* $\angle ABC \cong \angle DEF$




*Proof*

| Statements | Reasons |
|---|---|
| **1.** $\angle ABC$ and $\angle DEF$ are right angles. | **1.** Given. |
| **2.** $m\angle ABC = 90$ and $m\angle DEF = 90$ | **2.** Definition of right angle. |
| **3.** $m\angle ABC = m\angle DEF$ | **3.** Transitive property of equality. |
| **4.** $\angle ABC \cong \angle DEF$ | **4.** Definition of congruent angles. |

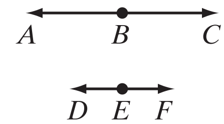
We can write this proof in paragraph form as follows:

*Proof:* A right angle is an angle whose degree measure is 90. Therefore, $m\angle ABC$ is 90 and $m\angle DEF$ is 90. Since $m\angle ABC$ and $m\angle DEF$ are both equal to the same quantity, they are equal to each other. Since $\angle ABC$ and $\angle DEF$ have equal measures, they are congruent.

**Theorem 4.2** If two angles are straight angles, then they are congruent.

*Given* $\angle ABC$ and $\angle DEF$ are straight angles.

*Prove* $\angle ABC \cong \angle DEF$

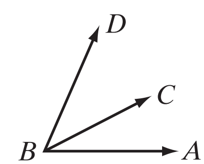



The proof of this theorem, which is similar to the proof of Theorem 4.1, is left to the student. (See exercise 17.)

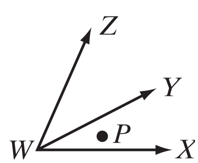
## Definitions Involving Pairs of Angles

**DEFINITION**

**Adjacent angles** are two angles in the same plane that have a common vertex and a common side but do not have any interior points in common.



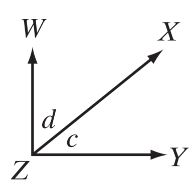

Angle $ABC$ and $\angle CBD$ are adjacent angles because they have $B$ as their common vertex, $\overrightarrow{BC}$ as their common side, and no interior points in common.

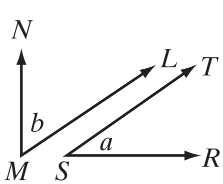



However, $\angle XWY$ and $\angle XWZ$ are not adjacent angles. Although $\angle XWY$ and $\angle XWZ$ have $W$ as their common vertex and $\overrightarrow{WX}$ as their common side, they have interior points in common. For example, point $P$ is in the interior of both $\angle XWY$ and $\angle XWZ$.

**DEFINITION**

**Complementary angles** are two angles, the sum of whose degree measures is 90.




Each angle is called the **complement** of the other. If $m\angle c = 40$ and $m\angle d = 50$, then $\angle c$ and $\angle d$ are complementary angles. If $m\angle a = 35$ and $m\angle b = 55$, then $\angle a$ and $\angle b$ are complementary angles.

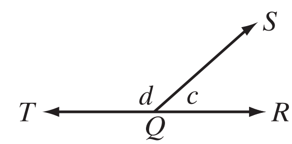
Complementary angles may be adjacent, as in the case of $\angle c$ and $\angle d$, or they may be nonadjacent, as in the case of $\angle a$ and $\angle b$. Note that if the two complementary angles are adjacent, their sum is a right angle: $\angle c + \angle d = \angle WZY$, a right angle.

Since $m\angle c + m\angle d = 90$, we say that $\angle c$ is the complement of $\angle d$, and that $\angle d$ is the complement of $\angle c$. When the degree measure of an angle is $k$, the degree measure of the complement of the angle is $(90 - k)$ because $k + (90 - k) = 90$.

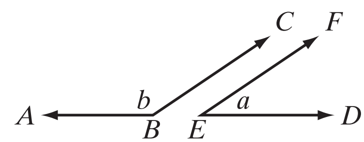
**DEFINITION**

**Supplementary angles** are two angles, the sum of whose degree measures is 180.

When two angles are supplementary, each angle is called the **supplement** of the other. If m$\angle c$ = 40 and m$\angle d$ = 140, then $\angle c$ and $\angle d$ are supplementary angles. If m$\angle a$ = 35 and m$\angle b$ = 145, then $\angle a$ and $\angle b$ are supplementary angles.




Supplementary angles may be adjacent, as in the case of $\angle c$ and $\angle d$, or they may be nonadjacent, as in the case of $\angle a$ and $\angle b$. Note that if the two supplementary angles are adjacent, their sum is a straight angle. Here $\angle c + \angle d = \angle TQR$, a straight angle.




Since m$\angle c$ + m$\angle d$ = 180, we say that $\angle c$ is the supplement of $\angle d$ and that $\angle d$ is the supplement of $\angle c$. When the degree measure of an angle is $k$, the degree measure of the supplement of the angle is $(180 - k)$ because $k + (180 - k) = 180$.

**EXAMPLE 1**

Find the measure of an angle if its measure is 24 degrees more than the measure of its complement.

***Solution*** Let $x$ = measure of complement of angle.

Then $x + 24$ = measure of angle.

*The sum of the degree measures of an angle and its complement is 90.*

$$\begin{aligned} x + x + 24 &= 90 \\ 2x + 24 &= 90 \\ 2x &= 66 \\ x &= 33 \\ x + 24 &= 57 \end{aligned}$$

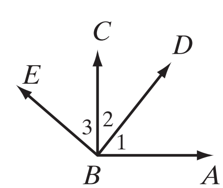
*Answer* The measure of the angle is 57 degrees.

## Theorems Involving Pairs of Angles

**Theorem 4.3** If two angles are complements of the same angle, then they are congruent.

*Given* $\angle 1$ is the complement of $\angle 2$ and $\angle 3$ is the complement of $\angle 2$.

*Prove* $\angle 1 \cong \angle 3$



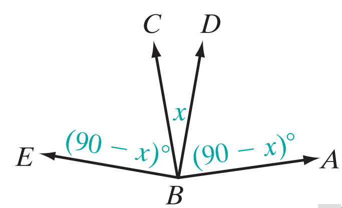
*Proof*

| Statements | Reasons |
|---|---|
| **1.** $\angle 1$ is the complement of $\angle 2$. | **1.** Given. |
| **2.** $m\angle 1 + m\angle 2 = 90$ | **2.** Complementary angles are two angles the sum of whose degree measures is 90. |
| **3.** $\angle 3$ is the complement of $\angle 2$. | **3.** Given. |
| **4.** $m\angle 3 + m\angle 2 = 90$ | **4.** Definition of complementary angles. |
| **5.** $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$ | **5.** Transitive property of equality (steps 2 and 4). |
| **6.** $m\angle 2 = m\angle 2$ | **6.** Reflexive property of equality. |
| **7.** $m\angle 1 = m\angle 3$ | **7.** Subtraction postulate. |
| **8.** $\angle 1 \cong \angle 3$ | **8.** Congruent angles are angles that have the same measure. |

**Note:** In a proof, there are two acceptable ways to indicate a definition as a reason. In reason 2 of the proof above, the definition of complementary angles is stated in its complete form. It is also acceptable to indicate this reason by the phrase "Definition of complementary angles," as in reason 4.

We can also give an algebraic proof for the theorem just proved.

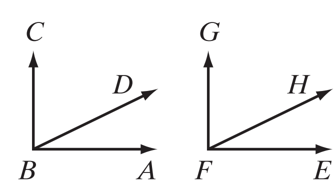
*Proof:* In the figure, $m\angle CBD = x$. Both $\angle ABD$ and $\angle CBE$ are complements to $\angle CBD$. Thus, $m\angle ABD = 90 - x$ and $m\angle CBE = 90 - x$, and we conclude $\angle ABD$ and $\angle CBE$ have the same measure. Since angles that have the same measure are congruent, $\angle ABD \cong \angle CBE$.



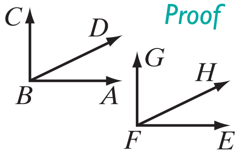

**Theorem 4.4** If two angles are congruent, then their complements are congruent.

*Given* $\angle ABD \cong \angle EFH$
$\angle CBD$ is the complement of $\angle ABD$.
$\angle GFH$ is the complement of $\angle EFH$.

*Prove* $\angle CBD \cong \angle GHF$




This theorem can be proved in a manner similar to Theorem 4.3 but with the use of the substitution postulate. We can also use an algebraic proof.

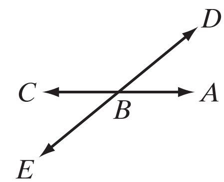



*Proof* Congruent angles have the same measure. If $\angle ABD \cong \angle EFH$, we can represent the measure of each angle by the same variable: $m\angle ABD = m\angle EFH = x$. Since $\angle CBD$ is the complement of $\angle ABD$, and $\angle GFH$ is the complement of $\angle EFH$, then $m\angle CBD = 90 - x$ and $m\angle GFH = 90 - x$. Therefore, $m\angle CBD = m\angle GFH$ and $\angle CBD \cong \angle GHF$.

**Theorem 4.5** If two angles are supplements of the same angle, then they are congruent.

*Given* $\angle ABD$ is the supplement of $\angle DBC$, and $\angle EBC$ is the supplement of $\angle DBC$.

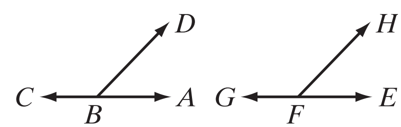
*Prove* $\angle ABD \cong \angle EBC$




**Theorem 4.6** If two angles are congruent, then their supplements are congruent.

*Given* $\angle ABD \cong \angle EFH$, $\angle CBD$ is the supplement of $\angle ABD$, and $\angle GFH$ is the supplement of $\angle EFH$.

*Prove* $\angle CBD \cong \angle GFH$

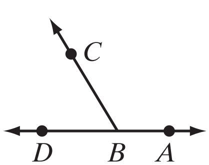



The proofs of Theorems 4.5 and 4.6 are similar to the proofs of Theorems 4.3 and 4.4 and will be left to the student. (See exercises 18 and 19.)

## More Definitions and Theorems Involving Pairs of Angles

**DEFINITION**

A **linear pair of angles** are two adjacent angles whose sum is a straight angle.

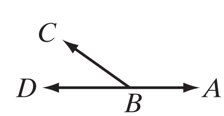



In the figure, $\angle ABD$ is a straight angle and $C$ is not on $\overleftrightarrow{ABD}$. Therefore, $\angle ABC + \angle CBD = \angle ABD$. Note that $\angle ABC$ and $\angle CBD$ are adjacent angles whose common side is $\overrightarrow{BC}$ and whose remaining sides are opposite rays that together form a straight line, $\overleftrightarrow{AD}$.

**Theorem 4.7** If two angles form a linear pair, then they are supplementary.

*Given* $\angle ABC$ and $\angle CBD$ form a linear pair.

*Prove* $\angle ABC$ and $\angle CBD$ are supplementary.



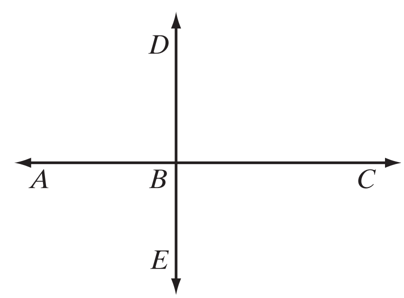
*Proof* In the figure, $\angle ABC$ and $\angle CBD$ form a linear pair. They share a common side, $\overrightarrow{BC}$, and their remaining sides, $\overrightarrow{BA}$ and $\overrightarrow{BD}$, are opposite rays. The sum of a linear pair of angles is a straight angle, and the degree measure of a straight angle is 180. Therefore, $m\angle ABC + m\angle CBD = 180$. Then, $\angle ABC$ and $\angle CBD$ are supplementary because supplementary angles are two angles the sum of whose degree measure is 180. ◻

**Theorem 4.8**

> If two lines intersect to form congruent adjacent angles, then they are perpendicular.

*Given* $\overleftrightarrow{ABC}$ and $\overleftrightarrow{DBE}$ with $\angle ABD \cong \angle DBC$

*Prove* $\overleftrightarrow{ABC} \perp \overleftrightarrow{DBE}$




*Proof* The union of the opposite rays, $\overrightarrow{BA}$ and $\overrightarrow{BC}$, is the straight angle, $\angle ABC$. The measure of straight angle is 180. By the partition postulate, $\angle ABC$ is the sum of $\angle ABD$ and $\angle DBC$. Thus,

$$\begin{aligned} m\angle ABD + m\angle DBC &= m\angle ABC \\ &= 180 \end{aligned}$$

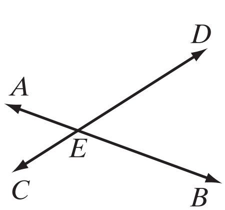
Since $\angle ABD \cong \angle DBC$, they have equal measures. Therefore,

$$m\angle ABD = m\angle DBC = \tfrac{1}{2}(180) = 90$$

The angles, $\angle ABD$ and $\angle DBC$, are right angles. Therefore, $\overleftrightarrow{ABC} \perp \overleftrightarrow{DBE}$ because perpendicular lines intersect to form right angles. ◻

**DEFINITION**

**Vertical angles** are two angles in which the sides of one angle are opposite rays to the sides of the second angle.

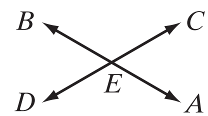



In the figure, $\angle AEC$ and $\angle DEB$ are a pair of vertical angles because $\overrightarrow{EA}$ and $\overrightarrow{EB}$ are opposite rays and $\overrightarrow{EC}$ and $\overrightarrow{ED}$ are opposite rays. Also, $\angle AED$ and $\angle CEB$ are a pair of vertical angles because $\overrightarrow{EA}$ and $\overrightarrow{EB}$ are opposite rays and $\overrightarrow{ED}$ and $\overrightarrow{EC}$ are opposite rays. In each pair of vertical angles, the opposite rays, which are the sides of the angles, form straight lines, $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. When two straight lines intersect, two pairs of vertical angles are formed.

**Theorem 4.9** If two lines intersect, then the vertical angles are congruent.

*Given* $\overleftrightarrow{AEB}$ and $\overleftrightarrow{CED}$ intersect at $E$.

*Prove* $\angle BEC \cong \angle AED$




*Proof*

| Statements | Reasons |
|---|---|
| **1.** $\overleftrightarrow{AEB}$ and $\overleftrightarrow{CED}$ intersect at $E$. | **1.** Given. |
| **2.** $\overrightarrow{EA}$ and $\overrightarrow{EB}$ are opposite rays. $\overrightarrow{ED}$ and $\overrightarrow{EC}$ are opposite rays. | **2.** Definition of opposite rays. |
| **3.** $\angle BEC$ and $\angle AED$ are vertical angles. | **3.** Definition of vertical angles. |
| **4.** $\angle BEC$ and $\angle AEC$ are a linear pair. $\angle AEC$ and $\angle AED$ are a linear pair. | **4.** Definition of a linear pair. |
| **5.** $\angle BEC$ and $\angle AEC$ are supplementary. $\angle AEC$ and $\angle AED$ are supplementary. | **5.** If two angles form a linear pair, they are supplementary. (Theorem 4.7) |
| **6.** $\angle BEC \cong \angle AED$ | **6.** If two angles are supplements of the same angle, they are congruent. (Theorem 4.5) |

In the proof above, reasons 5 and 6 demonstrate how previously proved theorems can be used as reasons in deducing statements in a proof. In this text, we have assigned numbers to theorems that we will use frequently in proving exercises as well as in proving other theorems. You do not need to remember the numbers of the theorems but you should memorize the statements of the theorems in order to use them as reasons when writing a proof. You may find it useful to keep a list of definitions, postulates, and theorems in a special section in your notebook or on index cards for easy reference and as a study aid.

In this chapter, we have seen the steps to be taken in presenting a proof in geometry using deductive reasoning:

**1.** As an aid, draw a figure that pictures the data of the theorem or the problem. Use letters to label points in the figure.

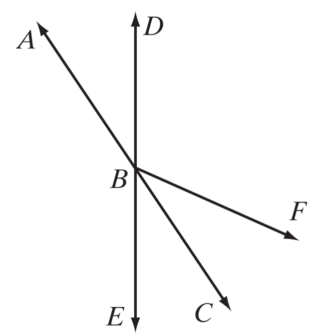
**2.** State the *given*, which is the hypothesis of the theorem, in terms of the figure.

**3.** State the *prove*, which is the conclusion of the theorem, in terms of the figure.

4. Present the *proof*, which is a series of logical arguments used in the demonstration. Each step in the proof should consist of a statement about the figure. Each statement should be justified by the given, a definition, a postulate, or a previously proved theorem. The proof may be presented in a two-column format or in paragraph form. Proofs that involve the measures of angles or of line segments can often be presented as an algebraic proof.

## EXAMPLE 2

If $\overleftrightarrow{ABC}$ and $\overleftrightarrow{DBE}$ intersect at $B$ and $\overrightarrow{BC}$ bisects $\angle EBF$, prove that $\angle CBF \cong \angle ABD$.




**Solution** *Given:* $\overleftrightarrow{ABC}$ and $\overleftrightarrow{DBE}$ intersect at $B$ and $\overrightarrow{BC}$ bisects $\angle EBF$.

*Prove:* $\angle CBF \cong \angle ABD$

*Proof*

| Statements | Reasons |
|---|---|
| **1.** $\overrightarrow{BC}$ bisects $\angle EBF$. | **1.** Given. |
| **2.** $\angle EBC \cong \angle CBF$ | **2.** Definition of a bisector of an angle. |
| **3.** $\overleftrightarrow{ABC}$ and $\overleftrightarrow{DBE}$ intersect at $B$. | **3.** Given. |
| **4.** $\angle EBC$ and $\angle ABD$ are vertical angles. | **4.** Definition of vertical angles. |
| **5.** $\angle EBC \cong \angle ABD$ | **5.** If two lines intersect, then the vertical angles are congruent. |
| **6.** $\angle CBF \cong \angle ABD$ | **6.** Transitive property of congruence (steps 2 and 5). |

*Alternative Proof* An algebraic proof can be given: Let $m\angle EBF = 2x$. It is given that $\overrightarrow{BC}$ bisects $\angle EBF$. The bisector of an angle separates the angle into two congruent angles: $\angle EBC$ and $\angle CBF$. Congruent angles have equal measures. Therefore, $m\angle EBC = m\angle CBF = x$.

It is also given that $\overleftrightarrow{ABC}$ and $\overleftrightarrow{DBE}$ intersect at $B$. If two lines intersect, the vertical angles are congruent and therefore have equal measures: $m\angle ABD = m\angle EBC = x$. Then since $m\angle CBF = x$ and $m\angle ABD = x$, $m\angle CBF = m\angle ABD$ and $\angle CBF \cong \angle ABD$.

## Exercises

### Writing About Mathematics

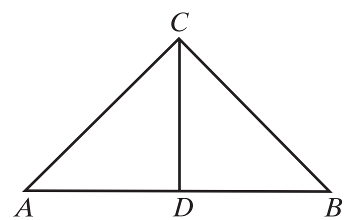
**1.** Josh said that Theorem 4.9 could also have been proved by showing that $\angle AEC \cong \angle BED$. Do you agree with Josh? Explain.

**2.** The statement of Theorem 4.7 is "If two angles form a linear pair then they are supplementary." Is the converse of this theorem true? Justify your answer.

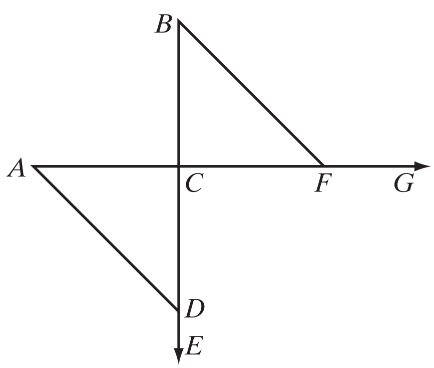
### Developing Skills

In 3–11, in each case write a proof, using the hypothesis as the given and the conclusion as the statement to be proved.

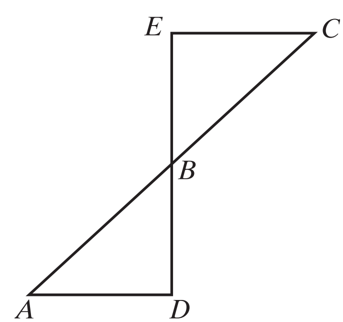
**3.** If $m\angle ACD + m\angle DCB = 90$, $\angle B \cong \angle DCA$, and $\angle A \cong \angle DCB$, then $\angle A$ and $\angle B$ are complements.



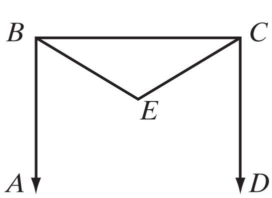

**4.** If $\overrightarrow{ACFG}$ and $\overrightarrow{BCDE}$ intersect at $C$ and $\angle ADC \cong \angle BFC$, then $\angle ADE \cong \angle BFG$.



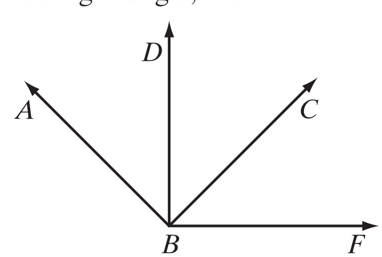

**5.** If $\angle ADB$ is a right angle and $\overline{CE} \perp \overline{DBE}$, then $\angle ADB \cong \angle CEB$.



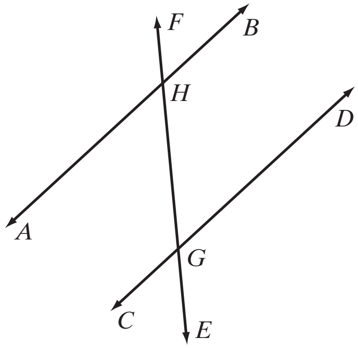

**6.** If $\angle ABC$ and $\angle BCD$ are right angles, and $\angle EBC \cong \angle ECB$, then $\angle EBA \cong \angle ECD$.



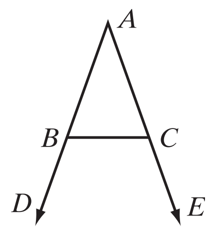

**7.** If $\angle ABC$ is a right angle and $\angle DBF$ is a right angle, then $\angle ABD \cong \angle CBF$.



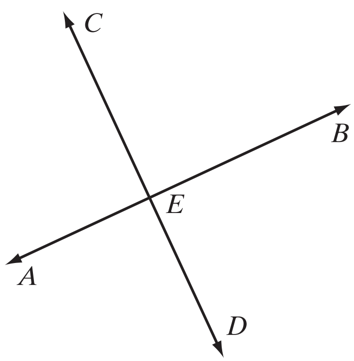
**8.** If $\overleftrightarrow{EF}$ intersects $\overleftrightarrow{AB}$ at $H$ and $\overleftrightarrow{DC}$ at $G$, and $m\angle BHG = m\angle CGH$, then $\angle BHG \cong \angle DGE$.



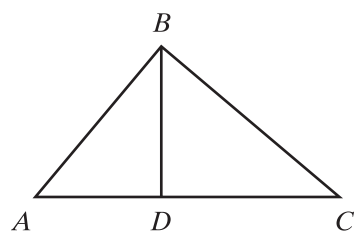

**9.** If $\overrightarrow{ABD}$ and $\overrightarrow{ACE}$ intersect at $A$, and $\angle ABC \cong \angle ACB$, then $\angle DBC \cong \angle ECB$.




**10.** If $\overleftrightarrow{AEB}$ and $\overleftrightarrow{CED}$ intersect at $E$, and $\angle AEC \cong \angle CEB$, then $\overleftrightarrow{AEB} \perp \overleftrightarrow{CED}$.




**11.** If $\angle ABC$ is a right angle, and $\angle BAC$ is complementary to $\angle DBA$, then $\angle BAC \cong \angle CBD$.



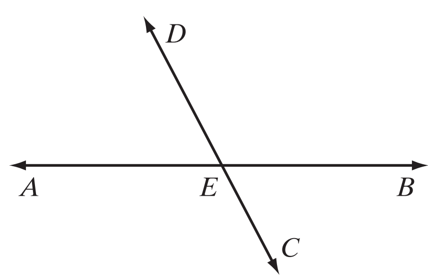

In 12–15, $\overleftrightarrow{AEB}$ and $\overleftrightarrow{CED}$ intersect at $E$.

**12.** If $m\angle BEC = 70$, find $m\angle AED$, $m\angle DEB$, and $m\angle AEC$.

**13.** If $m\angle DEB = 2x + 20$ and $m\angle AEC = 3x - 30$, find $m\angle DEB$, $m\angle AEC$, $m\angle AED$, and $m\angle CEB$.

**14.** If $m\angle BEC = 5x - 25$ and $m\angle DEA = 7x - 65$, find $m\angle BEC$, $m\angle DEA$, $m\angle DEB$, and $m\angle AEC$.

**15.** If $m\angle BEC = y$, $m\angle DEB = 3x$, and $m\angle DEA = 2x - y$, find $m\angle CEB$, $m\angle BED$, $m\angle DEA$, and $m\angle AEC$.



16. $\overleftrightarrow{RS}$ intersects $\overleftrightarrow{LM}$ at $P$, $m\angle RPL = x + y$, $m\angle LPS = 3x + 2y$, $m\angle MPS = 3x - 2y$.

    **a.** Solve for $x$ and $y$.

    **b.** Find $m\angle RPL$, $m\angle LPS$, and $m\angle MPS$.

17. Prove Theorem 4.2, "If two angles are straight angles, then they are congruent."

18. Prove Theorem 4.5, "If two angles are supplements of the same angle, then they are congruent."

19. Prove Theorem 4.6, "If two angles are congruent, then their supplements are congruent."

## Applying Skills

20. Two angles form a linear pair. The measure of the smaller angle is one-half the measure of the larger angle. Find the degree measure of the larger angle.

21. The measure of the supplement of an angle is 60 degrees more than twice the measure of the angle. Find the degree measure of the angle.

22. The difference between the degree measures of two supplementary angles is 80. Find the degree measure of the larger angle.

23. Two angles are complementary. The measure of the larger angle is 5 times the measure of the smaller angle. Find the degree measure of the larger angle.

24. Two angles are complementary. The degree measure of the smaller angle is 50 less than the degree measure of the larger. Find the degree measure of the larger angle.

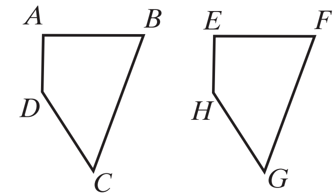
25. The measure of the complement of an angle exceeds the measure of the angle by 24 degrees. Find the degree measure of the angle.

# 4-4 CONGRUENT POLYGONS AND CORRESPONDING PARTS

Fold a rectangular sheet of paper in half by placing the opposite edges together. If you tear the paper along the fold, you will have two rectangles that fit exactly on one another. We call these rectangles *congruent polygons*. **Congruent polygons** are polygons that have the same size and shape. Each angle of one polygon is congruent to an angle of the other and each edge of one polygon is congruent to an edge of the other.

In the diagram at the top of page 155, polygon $ABCD$ is congruent to polygon $EFGH$. Note that the congruent polygons are named in such a way that each vertex of $ABCD$ corresponds to exactly one vertex of $EFGH$ and each vertex of $EFGH$ corresponds to exactly one vertex of $ABCD$. This relationship is called a **one-to-one correspondence**. The order in which the vertices are named shows this one-to-one correspondence of points.

$ABCD \cong EFGH$ indicates that:




- $A$ corresponds to $E$; $E$ corresponds to $A$.
- $B$ corresponds to $F$; $F$ corresponds to $B$.
- $C$ corresponds to $G$; $G$ corresponds to $C$.
- $D$ corresponds to $H$; $H$ corresponds to $D$.

Congruent polygons should always be named so as to indicate the correspondences between the vertices of the polygons.

## Corresponding Parts of Congruent Polygons

In congruent polygons $ABCD$ and $EFGH$ shown above, vertex $A$ corresponds to vertex $E$. Angles $A$ and $E$ are called **corresponding angles**, and $\angle A \cong \angle E$.

In this example, there are four pairs of such corresponding angles:

$$\angle A \cong \angle E \quad \angle B \cong \angle F \quad \angle C \cong \angle G \quad \angle D \cong \angle H$$

In congruent polygons, corresponding angles are congruent.

In congruent polygons $ABCD$ and $EFGH$, since $A$ corresponds to $E$ and $B$ corresponds to $F$, $\overline{AB}$ and $\overline{EF}$ are **corresponding sides**, and $\overline{AB} \cong \overline{EF}$.

In this example, there are four pairs of such corresponding sides:

$$\overline{AB} \cong \overline{EF} \quad \overline{BC} \cong \overline{FG} \quad \overline{CD} \cong \overline{GH} \quad \overline{DA} \cong \overline{HE}$$

In congruent polygons, corresponding sides are congruent.

The pairs of congruent angles and the pairs of congruent sides are called the corresponding parts of congruent polygons. We can now present the formal definition for congruent polygons.

**DEFINITION**

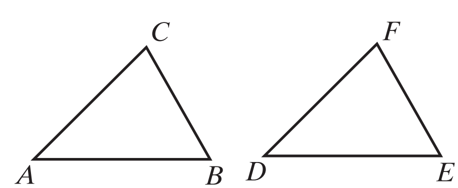
Two **polygons are congruent** if and only if there is a one-to-one correspondence between their vertices such that corresponding angles are congruent and corresponding sides are congruent.

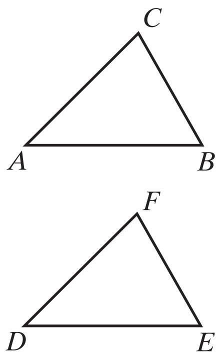
This definition can be stated more simply as follows:

► **Corresponding parts of congruent polygons are congruent.**

## Congruent Triangles

The smallest number of sides that a polygon can have is three. A *triangle* is a polygon with exactly three sides. In the figure, $\triangle ABC$ and $\triangle DEF$ are congruent triangles.






The correspondence establishes six facts about these triangles: three facts about corresponding sides and three facts about corresponding angles. In the table at the right, these six facts are stated as equalities. Since each congruence statement is equivalent to an equality statement, we will use whichever notation serves our purpose better in a particular situation.

| Congruences | Equalities |
|---|---|
| $\overline{AB} \cong \overline{DE}$ | $AB = DE$ |
| $\overline{BC} \cong \overline{EF}$ | $BC = EF$ |
| $\overline{AC} \cong \overline{DF}$ | $AC = DF$ |
| $\angle A \cong \angle D$ | $m\angle A = m\angle D$ |
| $\angle B \cong \angle E$ | $m\angle B = m\angle E$ |
| $\angle C \cong \angle F$ | $m\angle C = m\angle F$ |

For example, in one proof, we may prefer to write $\overline{AC} \cong \overline{DF}$ and in another proof to write $AC = DF$. In the same way, we might write $\angle C \cong \angle F$ or we might write $m\angle C = m\angle F$. From the definition, we may now say:

► **Corresponding parts of congruent triangles are equal in measure.**

In two congruent triangles, pairs of corresponding sides are always opposite pairs of corresponding angles. In the preceding figure, $\triangle ABC \cong \triangle DEF$. The order in which we write the names of the vertices of the triangles indicates the one-to-one correspondence.

1. $\angle A$ and $\angle D$ are corresponding congruent angles.
2. $\overline{BC}$ is opposite $\angle A$, and $\overline{EF}$ is opposite $\angle D$.
3. $\overline{BC}$ and $\overline{EF}$ are corresponding congruent sides.

## Equivalence Relation of Congruence

In Section 3-5 we saw that the relation "is congruent to" is an equivalence relation for the set of line segments and the set of angles. Therefore, "is congruent to" must be an equivalence relation for the set of triangles or the set of polygons with a given number of sides.

1. Reflexive property: $\triangle ABC \cong \triangle ABC$.
2. Symmetric property: If $\triangle ABC \cong \triangle DEF$, then $\triangle DEF \cong \triangle ABC$.
3. Transitive property: If $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle RST$, then $\triangle ABC \cong \triangle RST$.

Therefore, we state these properties of congruence as three postulates:

**Postulate 4.11** Any geometric figure is congruent to itself. (Reflexive Property)

**Postulate 4.12** A congruence may be expressed in either order. (Symmetric Property)

**Postulate 4.13** Two geometric figures congruent to the same geometric figure are congruent to each other. (Transitive Property)

## Exercises

### Writing About Mathematics

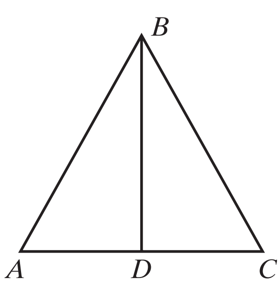
**1.** If $\triangle ABC \cong \triangle DEF$, then $\overline{AB} \cong \overline{DE}$. Is the converse of this statement true? Justify your answer.

**2.** Jesse said that since $\triangle RST$ and $\triangle STR$ name the same triangle, it is correct to say $\triangle RST \cong \triangle STR$. Do you agree with Jesse? Justify your answer.

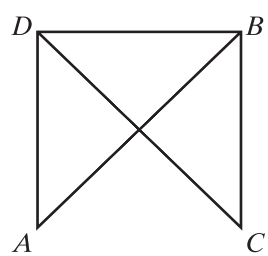
### Developing Skills

In 3–5, in each case name three pairs of corresponding angles and three pairs of corresponding sides in the given congruent triangles. Use the symbol $\cong$ to indicate that the angles named and also the sides named in your answers are congruent.

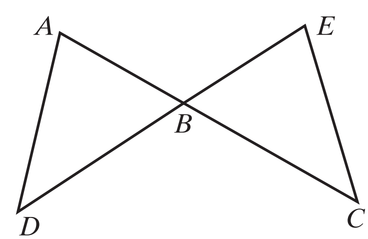
**3.** $\triangle ABD \cong \triangle CBD$



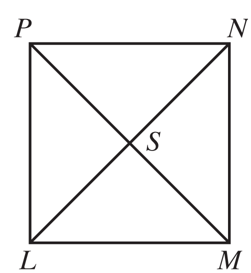

**4.** $\triangle ADB \cong \triangle CBD$




**5.** $\triangle ABD \cong \triangle EBC$




In 6–10, $LMNP$ is a square and $\overline{LSN}$ and $\overline{PSM}$ bisect each other. Name the property that justifies each statement.




**6.** $\triangle LSP \cong \triangle LSP$

**7.** If $\triangle LSP \cong \triangle NSM$, then $\triangle NSM \cong \triangle LSP$.

**8.** If $\triangle LSP \cong \triangle NSM$ and $\triangle NSM \cong \triangle NSP$, then $\triangle LSP \cong \triangle NSP$.

**9.** If $LS = SN$, then $SN = LS$.

**10.** If $\angle PLM \cong \angle PNM$, then $\angle PNM \cong \angle PLM$.

## 4-5 PROVING TRIANGLES CONGRUENT USING SIDE, ANGLE, SIDE

The definition of congruent polygons states that two polygons are congruent if and only if each pair of corresponding sides and each pair of corresponding angles are congruent. However, it is possible to prove two triangles congruent by proving that fewer than three pairs of sides and three pairs of angles are congruent.

### Hands-On Activity

In this activity, we will use a protractor and ruler, or geometry software.

Use the procedure below to draw a triangle given the measures of two sides and of the included angle.

**STEP 1.** Use the protractor or geometry software to draw an angle with the given measure.

**STEP 2.** Draw two segments along the rays of the angle with the given lengths. The two segments should share the vertex of the angle as a common endpoint.

**STEP 3.** Join the endpoints of the two segments to form a triangle.

**STEP 4.** Repeat steps 1 through 3 to draw a second triangle using the same angle measure and segment lengths.

**a.** Follow the steps to draw two different triangles with each of the given side-angle-side measures.

(1) 3 in., 90°, 4 in.
(2) 5 in., 40°, 5 in.
(3) 5 cm, 115°, 8 cm
(4) 10 cm, 30°, 8 cm

**b.** For each pair of triangles, measure the side and angles that were not given. Do they have equal measures?

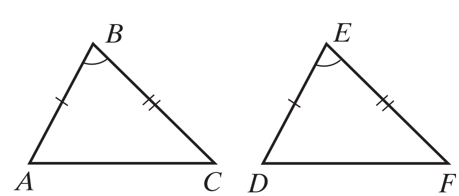
**c.** Are the triangles of each pair congruent? Does it appear that when two sides and the included angle of one triangle are congruent to the corresponding sides and angle of another, that the triangles are congruent?

This activity leads to the following statement of side-angle-side or **SAS triangle congruence**, whose truth will be assumed without proof:

**Postulate 4.14**

> Two triangles are congruent if two sides and the included angle of one triangle are congruent, respectively, to two sides and the included angle of the other. (SAS)

In $\triangle ABC$ and $\triangle DEF$, $\overline{BA} \cong \overline{ED}$, $\angle ABC \cong \angle DEF$ and $\overline{BC} \cong \overline{EF}$. It follows that $\triangle ABC \cong \triangle DEF$. The postulate used here is abbreviated SAS.



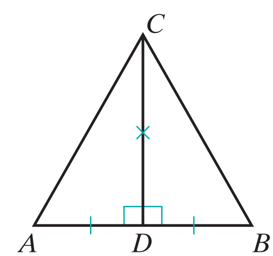
**Note:** When congruent sides and angles are listed, a correspondence is established. Since the vertices of congruent angles, $\angle ABC$ and $\angle DEF$, are $B$ and $E$, $B$ corresponds to $E$. Since $BA = ED$ and $B$ corresponds to $E$, then $A$ corresponds to $D$, and since $BC = EF$ and $B$ corresponds to $E$, then $C$ corresponds to $F$. We can write $\triangle ABC \cong \triangle DEF$. But, when naming the triangle, if we change the order of the vertices in one triangle we must change the order in the other. For example, $\triangle ABC$, $\triangle BAC$, and $\triangle CBA$ name the same triangle. If $\triangle ABC \cong \triangle DEF$, we may write $\triangle BAC \cong \triangle EDF$ or $\triangle CBA \cong \triangle FED$, but we may not write $\triangle ABC \cong \triangle EFD$.

## EXAMPLE 1

*Given*: $\triangle ABC$, $\overline{CD}$ is the bisector of $\overline{AB}$, and $\overline{CD} \perp \overline{AB}$.

*Prove*: $\triangle ACD \cong \triangle BCD$

Prove the triangles congruent by SAS.




**Proof**

| | **Statements** | **Reasons** |
|---|---|---|
| | **1.** $\overline{CD}$ bisects $\overline{AB}$. | **1.** Given. |
| | **2.** $D$ is the midpoint of $\overline{AB}$. | **2.** The bisector of a line segment intersects the segment at its midpoint. |
| S | **3.** $\overline{AD} \cong \overline{DB}$ | **3.** The midpoint of a line segment divides the segment into two congruent segments. |
| | **4.** $\overline{CD} \perp \overline{AB}$ | **4.** Given. |
| | **5.** $\angle ADC$ and $\angle BDC$ are right angles. | **5.** Perpendicular lines intersect to form right angles. |
| A | **6.** $\angle ADC \cong \angle BDC$ | **6.** If two angles are right angles, then they are congruent. |
| S | **7.** $\overline{CD} \cong \overline{CD}$ | **7.** Reflexive property of congruence. |
| | **8.** $\triangle ACD \cong \triangle BCD$ | **8.** SAS (steps 3, 6, 7). |

Note that it is often helpful to mark segments and angles that are congruent with the same number of strokes or arcs. For example, in the diagram for this proof, $\overline{AD}$ and $\overline{DB}$ are marked with a single stroke, $\angle ADC$ and $\angle BDC$ are marked with the symbol for right angles, and $\overline{CD}$ is marked with an "×" to indicate a side common to both triangles.

## Exercises

### Writing About Mathematics

1. Each of two telephone poles is perpendicular to the ground and braced by a wire that extends from the top of the pole to a point on the level ground 5 feet from the foot of the pole. The wires used to brace the poles are of unequal lengths. Is it possible for the telephone poles to be of equal height? Explain your answer.

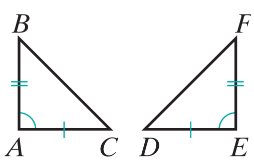
2. Is the following statement true? If two triangles are not congruent, then each pair of corresponding sides and each pair of corresponding angles are not congruent. Justify your answer.

### Developing Skills

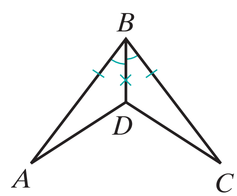
In 3–8, pairs of line segments marked with the same number of strokes are congruent. Pairs of angles marked with the same number of arcs are congruent. A line segment or an angle marked with an "×" is congruent to itself by the reflexive property of congruence.

In each case, is the given information sufficient to prove congruent triangles using SAS?

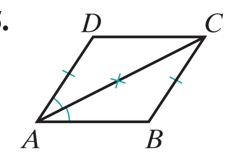
**3.**



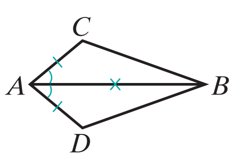

**4.**



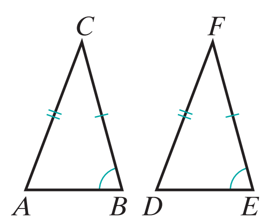

**5.**



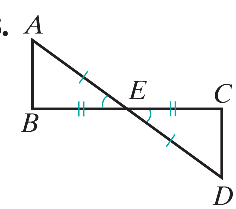

**6.**




**7.**

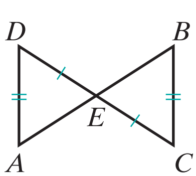



**8.**

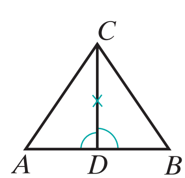



In 9–11, two sides or a side and an angle are marked to indicate that they are congruent. Name the pair of corresponding sides or corresponding angles that would have to be proved congruent in order to prove the triangles congruent by SAS.

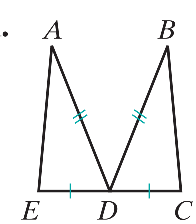
**9.**




**10.**




**11.**



### Applying Skills

In 12–14:

**a.** Draw a diagram with geometry software or pencil and paper and write a *given* statement using the information in the first sentence.

**b.** Use the information in the second sentence to write a *prove* statement.

**c.** Write a proof.

**12.** $\overline{ABC}$ and $\overline{DBE}$ bisect each other. Prove that $\triangle ABE \cong \triangle CBD$.

**13.** $ABCD$ is a quadrilateral; $AB = CD$; $BC = DA$; and $\angle DAB$, $\angle ABC$, $\angle BCD$, and $\angle CDA$ are right angles. Prove that the diagonal $\overline{AC}$ separates the quadrilateral into two congruent triangles.

**14.** $\angle PQR$ and $\angle RQS$ are a linear pair of angles that are congruent and $PQ = QS$. Prove that $\triangle PQR \cong \triangle RQS$.

## 4-6 PROVING TRIANGLES CONGRUENT USING ANGLE, SIDE, ANGLE

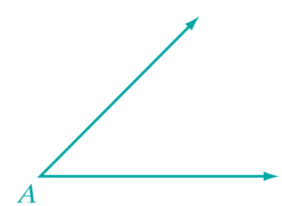
In the last section we saw that it is possible to prove two triangles congruent by proving that fewer than three pairs of sides and three pairs of angles are congruent, that is, by proving that two sides and the included angle of one triangle are congruent to the corresponding parts of another. There are also other ways of proving two triangles congruent.

### Hands-On Activity

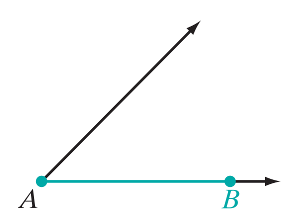
In this activity, we will use a protractor and ruler, or geometry software.

Use the procedure below to draw a triangle given the measures of two angles and of the included side.

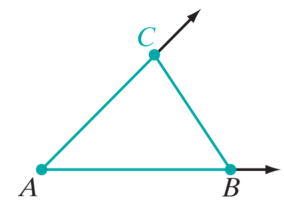
**STEP 1.** Use the protractor or geometry software to draw an angle with the first given angle measure. Call the vertex of that angle $A$.




**STEP 2.** Draw a segment with the given length along one of the rays of $\angle A$. One of the endpoints of the segment should be $A$. Call the other endpoint $B$.

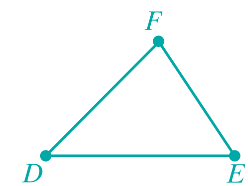


**STEP 3.** Draw a second angle of the triangle using the other given angle measure. Let $B$ be the vertex of this second angle and let $\overrightarrow{BA}$ be one of the sides of this angle.




**STEP 4.** Let $C$ be the intersection of the rays of $\angle A$ and $\angle B$ that are not on $\overleftrightarrow{AB}$.

**STEP 5.** Repeat steps 1 through 4. Let $D$ the vertex of the first angle, $E$ the vertex of the second angle, and $F$ the intersection of the rays of $\angle D$ and $\angle E$.




**a.** Follow the steps to draw *two* different triangles with each of the given angle-side-angle measures.
(1) 65°, 4 in., 35°
(2) 60°, 4 in., 60°
(3) 120°, 9 cm, 30°
(4) 30°, 9 cm, 30°

**b.** For each pair of triangles, measure the angle and sides that were not given. Do they have equal measures?

**c.** For each pair of triangles, you can conclude that the triangles are congruent.

The triangles formed, $\triangle ABC$ and $\triangle DEF$, can be placed on top of one another so that $A$ and $D$ coincide, $B$ and $E$ coincide, and $C$ and $F$ coincide. Therefore, $\triangle ABC \cong \triangle DEF$.

This activity leads to the following statement of angle-side-angle or **ASA triangle congruence**, whose truth will be assumed without proof:

**Postulate 4.15**

> Two triangles are congruent if two angles and the included side of one triangle are congruent, respectively, to two angles and the included side of the other. (ASA)

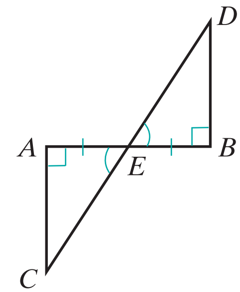
Thus, in $\triangle ABC$ and $\triangle DEF$, if $\angle B \cong \angle E$, $\overline{BA} \cong \overline{ED}$, and $\angle A \cong \angle D$, it follows that $\triangle ABC \cong \triangle DEF$. The postulate used here is abbreviated ASA. We will now use this postulate to prove two triangles congruent.

### EXAMPLE 1

*Given*: $\overline{AEB}$ and $\overline{CED}$ intersect at $E$, $E$ is the midpoint of $\overline{AEB}$, $\overline{AC} \perp \overline{AE}$, and $\overline{BD} \perp \overline{BE}$.

*Prove*: $\triangle AEC \cong \triangle BDE$

Prove the triangles congruent by ASA.




**Proof**

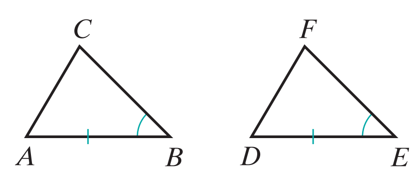
| | Statements | Reasons |
|---|---|---|
| | 1. $\overleftrightarrow{AEB}$ and $\overleftrightarrow{CED}$ intersect at $E$. | 1. Given. |
| A | 2. $\angle AEC \cong \angle BED$ | 2. If two lines intersect, the vertical angles are congruent. |
| | 3. $E$ is the midpoint of $\overline{AEB}$. | 3. Given. |
| S | 4. $\overline{AE} \cong \overline{BE}$ | 4. The midpoint of a line segment divides the segment into two congruent segments. |
| | 5. $\overline{AC} \perp \overline{AE}$ and $\overline{BD} \perp \overline{BE}$ | 5. Given. |
| | 6. $\angle A$ and $\angle B$ are right angles. | 6. Perpendicular lines intersect to form right angles. |
| A | 7. $\angle A \cong \angle B$ | 7. If two angles are right angles, they are congruent. |
| | 8. $\triangle AEC \cong \triangle BED$ | 8. ASA (steps 2, 4, 7). |

## Exercises

### Writing About Mathematics

1. Marty said that if two triangles are congruent and one of the triangles is a right triangle, then the other triangle must be a right triangle. Do you agree with Marty? Explain why or why not.

2. In $\triangle ABC$ and $\triangle DEF$, $\overline{AB} \cong \overline{DE}$, and $\angle B \cong \angle E$.



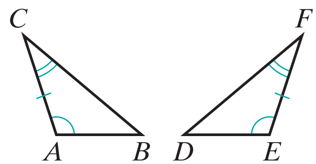

   a. Dora said that if $\overline{AC}$ is congruent to $\overline{DF}$, then it follows that $\triangle ABC \cong \triangle DEF$. Do you agree with Dora? Justify your answer.

   b. Using only the SAS and ASA postulates, if $\triangle ABC$ is not congruent to $\triangle DEF$, what sides and what angles cannot be congruent?

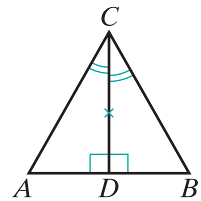
## Developing Skills

In 3–5, tell whether the triangles in each pair can be proved congruent by ASA, using only the marked congruent parts in establishing the congruence. Give the reason for your answer.

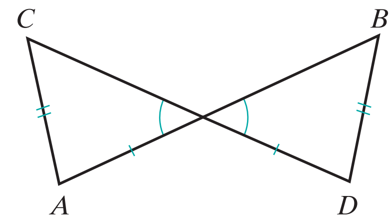
**3.**




**4.**

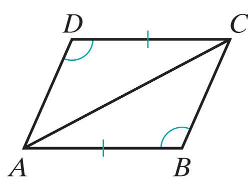



**5.**




In 6–8, in each case name the pair of corresponding sides or the pair of corresponding angles that would have to be proved congruent (in addition to those pairs marked congruent) in order to prove that the triangles are congruent by ASA.

**6.**

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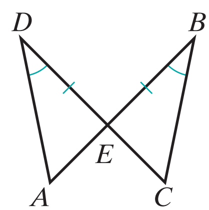


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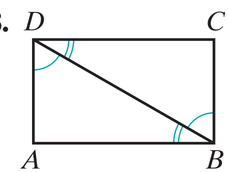
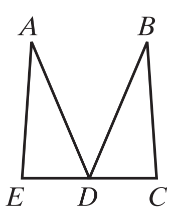



## Applying Skills

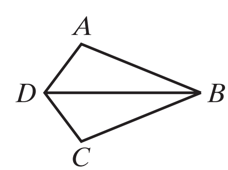
**9.** *Given*: $\angle E \cong \angle C$, $\angle EDA \cong \angle CDB$, and $D$ is the midpoint of $\overline{EC}$.

*Prove*: $\triangle DAE \cong \triangle DBC$



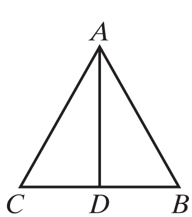

**10.** *Given*: $\overrightarrow{DB}$ bisects $\angle ADC$ and $\overrightarrow{BD}$ bisects $\angle ABC$.

*Prove*: $\triangle ABD \cong \triangle CBD$



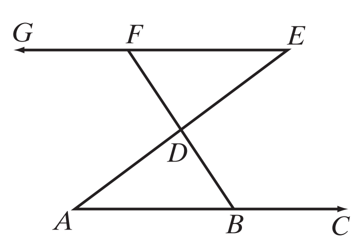

**11.** *Given*: $\overline{AD} \perp \overline{BC}$ and $\overrightarrow{AD}$ bisects $\angle BAC$.

*Prove*: $\triangle ADC \cong \triangle ADB$




**12.** *Given:* $\angle DBC \cong \angle GFD$ and $\overline{AE}$ bisects $\overline{FB}$ at $D$.

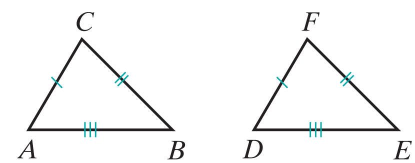
*Prove:* $\triangle DFE \cong \triangle DBA$



## 4-7 PROVING TRIANGLES CONGRUENT USING SIDE, SIDE, SIDE

Cut three straws to any lengths and put their ends together to form a triangle. Now cut a second set of straws to the same lengths and try to form a *different* triangle. Repeated experiments lead to the conclusion that it cannot be done.

As shown in $\triangle ABC$ and $\triangle DEF$, when all three pairs of corresponding sides of a triangle are congruent, the triangles must be congruent. The truth of this statement of side-side-side or **SSS triangle congruence** is assumed without proof.




**Postulate 4.16**

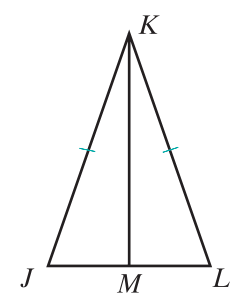
> Two triangles are congruent if the three sides of one triangle are congruent, respectively, to the three sides of the other. (SSS)

Thus, in $\triangle ABC$ and $\triangle DEF$ above, $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $\overline{BC} \cong \overline{EF}$. It follows that $\triangle ABC \cong \triangle DEF$. The postulate used here is abbreviated SSS.

### EXAMPLE 1

*Given:* Isosceles $\triangle JKL$ with $\overline{JK} \cong \overline{KL}$ and $M$ the midpoint of $\overline{JL}$.

*Prove:* $\triangle JKM \cong \triangle LKM$




***Proof*** Prove the triangles congruent by using SSS.

| *Statements* | *Reasons* |
|---|---|
| S **1.** $\overline{JK} \cong \overline{KL}$ | **1.** Given. |
| **2.** $M$ is the midpoint of $\overline{JL}$. | **2.** Given. |
| S **3.** $\overline{JM} \cong \overline{LM}$ | **3.** Definition of midpoint. |
| S **4.** $\overline{KM} \cong \overline{KM}$ | **4.** Reflexive property of congruence. |
| **5.** $\triangle JKM \cong \triangle LKM$ | **5.** SSS (steps 1, 3, 4). |

## Exercises

### Writing About Mathematics

1. Josh said that if two triangles are congruent and one of the triangles is isosceles, then the other must be isosceles. Do you agree with Josh? Explain why or why not.

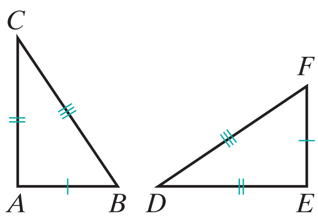
2. Alvan said that if two triangles are not congruent, then at least one of the three sides of one triangle is not congruent to the corresponding side of the other triangle. Do you agree with Alvan? Justify your answer.

### Developing Skills

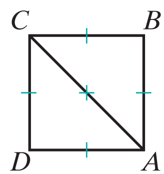
In 3–5, pairs of line segments marked with the same number of strokes are congruent. A line segment marked with "×" is congruent to itself by the reflexive property of congruence.

In each case, is the given information sufficient to prove congruent triangles?

**3.**




**4.**

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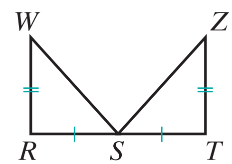
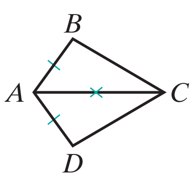



In 6–8, two sides are marked to indicate that they are congruent. Name the pair of corresponding sides that would have to be proved congruent in order to prove the triangles congruent by SSS.

**6.**

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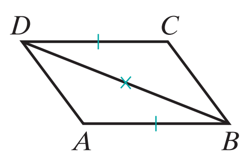


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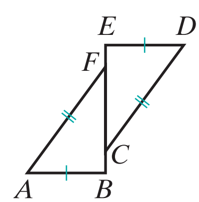
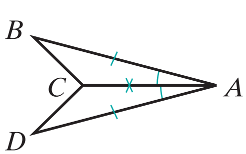



In 9–14, pairs of line segments marked with the same number of strokes are congruent. Pairs of angles marked with the same number of arcs are congruent. A line segment or an angle marked with "×" is congruent to itself by the reflexive property of congruence.

In each case, is the given information sufficient to prove congruent triangles? If so, write the abbreviation for the postulate that proves the triangles congruent.

**9.**

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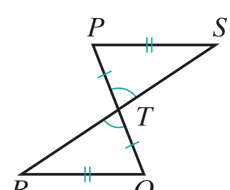
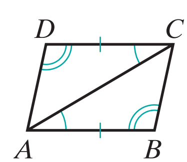
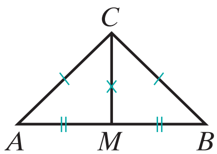
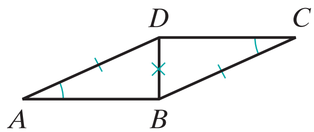
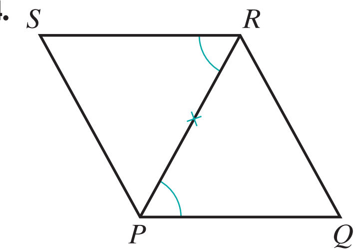



**11.**



12. 


13. 


14. 


**15.** If two sides and the angle opposite one of those sides in a triangle are congruent to the corresponding sides and angle of another triangle, are the two triangles congruent? Justify your answer or draw a counterexample proving that they are not.
(*Hint:* Could one triangle be an acute triangle and the other an obtuse triangle?)

## Applying Skills

**16.** *Given*: $\overline{AEB}$ bisects $\overline{CED}$, $\overline{AC} \perp \overline{CED}$, and $\overline{BD} \perp \overline{CED}$.
*Prove*: $\triangle EAC \cong \triangle EBD$

**17.** *Given*: $\triangle ABC$ is equilateral, $D$ is the midpoint of $\overline{AB}$.
*Prove*: $\triangle ACD \cong \triangle BCD$

**18.** *Given:* Triangle $PQR$ with $S$ on $\overline{PQ}$ and $\overline{RS} \perp \overline{PQ}$; $\triangle PSR$ is not congruent to $\triangle QSR$.
*Prove*: $PS \neq QS$

**19.** Gina is drawing a pattern for a kite. She wants it to consist of two congruent triangles that share a common side. She draws an angle with its vertex at $A$ and marks two points, $B$ and $C$, one on each of the rays of the angle. Each point, $B$ and $C$, is 15 inches from the vertex of the angle. Then she draws the bisector of $\angle BAC$, marks a point $D$ on the angle bisector and draws $\overline{BD}$ and $\overline{CD}$. Prove that the triangles that she drew are congruent.

## CHAPTER SUMMARY

***Definitions to Know***

- **Adjacent angles** are two angles in the same plane that have a common vertex and a common side but do not have any interior points in common.
- **Complementary angles** are two angles the sum of whose degree measures is 90.
- **Supplementary angles** are two angles the sum of whose degree measures is 180.
- A **linear pair of angles** are two adjacent angles whose sum is a straight angle.
- **Vertical angles** are two angles in which the sides of one angle are opposite rays to the sides of the second angle.

- Two **polygons are congruent** if and only if there is a one-to-one correspondence between their vertices such that corresponding angles are congruent and corresponding sides are congruent.
  - Corresponding parts of congruent polygons are congruent.
  - Corresponding parts of congruent polygons are equal in measure.

***Postulates***

**4.1** A line segment can be extended to any length in either direction.
**4.2** Through two given points, one and only one line can be drawn. (Two points determine a line.)
**4.3** Two lines cannot intersect in more than one point.
**4.4** One and only one circle can be drawn with any given point as center and the length of any given line segment as a radius.
**4.5** At a given point on a given line, one and only one perpendicular can be drawn to the line.
**4.6** From a given point not on a given line, one and only one perpendicular can be drawn to the line.
**4.7** For any two distinct points, there is only one positive real number that is the length of the line segment joining the two points. (Distance Postulate)
**4.8** The shortest distance between two points is the length of the line segment joining these two points.
**4.9** A line segment has one and only one midpoint.
**4.10** An angle has one and only one bisector.
**4.11** Any geometric figure is congruent to itself. (Reflexive Property)
**4.12** A congruence may be expressed in either order. (Symmetric Property)
**4.13** Two geometric figures congruent to the same geometric figure are congruent to each other. (Transitive Property)
**4.14** Two triangles are congruent if two sides and the included angle of one triangle are congruent, respectively, to two sides and the included angle of the other. (SAS)
**4.15** Two triangles are congruent if two angles and the included side of one triangle are congruent, respectively, to two angles and the included side of the other. (ASA)
**4.16** Two triangles are congruent if the three sides of one triangle are congruent, respectively, to the three sides of the other. (SSS)

***Theorems***

**4.1** If two angles are right angles, then they are congruent.
**4.2** If two angles are straight angles, then they are congruent.
**4.3** If two angles are complements of the same angle, then they are congruent.
**4.4** If two angles are congruent, then their complements are congruent.
**4.5** If two angles are supplements of the same angle, then they are congruent.
**4.6** If two angles are congruent, then their supplements are congruent.
**4.7** If two angles form a linear pair, then they are supplementary.
**4.8** If two lines intersect to form congruent adjacent angles, then they are perpendicular.
**4.9** If two lines intersect, then the vertical angles are congruent.

## VOCABULARY

**4-1** Distance postulate

**4-3** Adjacent angles • Complementary angles • Complement • Supplementary angles • Supplement • Linear pair of angles • Vertical angles

**4-4** One-to-one correspondence • Corresponding angles • Corresponding sides • Congruent polygons

**4-5** SAS triangle congruence

**4-6** ASA triangle congruence

**4-7** SSS triangle congruence

## REVIEW EXERCISES

**1.** The degree measure of an angle is 15 more than twice the measure of its complement. Find the measure of the angle and its complement.

**2.** Two angles, $\angle LMP$ and $\angle PMN$, are a linear pair of angles. If the degree measure of $\angle LMP$ is 12 less than three times that of $\angle PMN$, find the measure of each angle.

**3.** Triangle $JKL$ is congruent to triangle $PQR$ and $m\angle K = 3a + 18$ and $m\angle Q = 5a - 12$. Find the measure of $\angle K$ and of $\angle Q$.

**4.** If $\overline{ABC}$ and $\overline{DBE}$ intersect at $F$, what is true about $B$ and $F$? State a postulate that justifies your answer.

**5.** If $\overleftrightarrow{LM} \perp \overline{MN}$ and $\overleftrightarrow{KM} \perp \overline{MN}$, what is true about $\overleftrightarrow{LM}$ and $\overleftrightarrow{KM}$? State a postulate that justifies your answer.

**6.** Point $R$ is not on $\overline{LMN}$. Is $LM + MN$ less than, equal to, or greater than $LR + RN$? State a postulate that justifies your answer.

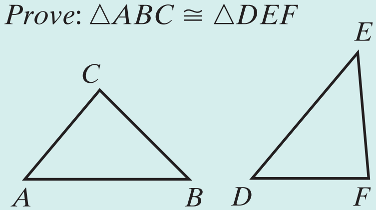
**7.** If $\overrightarrow{BD}$ and $\overrightarrow{BE}$ are bisectors of $\angle ABC$, does $E$ lie on $\overleftrightarrow{BD}$? State a postulate that justifies your answer.

**8.** The midpoint of $\overline{AB}$ is $M$. If $\overleftrightarrow{MN}$ and $\overleftrightarrow{PM}$ are bisectors of $\overline{AB}$, does $P$ lie on $\overleftrightarrow{MN}$? Justify your answer.

**9.** The midpoint of $\overline{AB}$ is $M$. If $\overleftrightarrow{MN}$ and $\overleftrightarrow{PM}$ are perpendicular to $\overline{AB}$, does $P$ lie on $\overleftrightarrow{MN}$? Justify your answer.

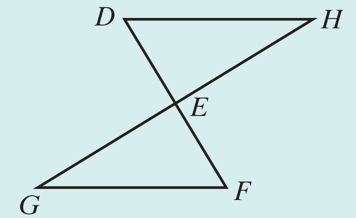
**10.** *Given*: $m\angle A = 50$, $m\angle B = 45$, $AB = 10$ cm, $m\angle D = 50$, $m\angle E = 45$, and $DE = 10$ cm.

*Prove*: $\triangle ABC \cong \triangle DEF$



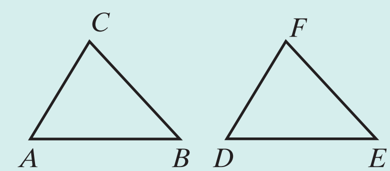

**11.** *Given*: $\overline{GEH}$ bisects $\overline{DEF}$ and $m\angle D = m\angle F$.

*Prove*: $\triangle GFE \cong \triangle HDE$




**12.** *Given:* $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\triangle ABC$ is not congruent to $\triangle DEF$.

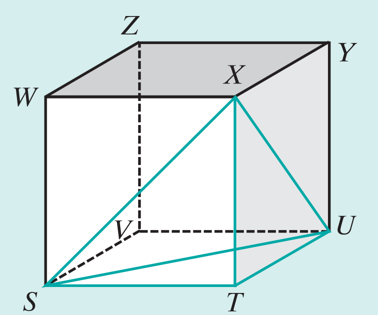
*Prove:* $\angle B$ is not congruent to $\angle E$.




## Exploration

1. If three angles of one triangle are congruent to the corresponding angles of another triangle, the triangles may or may not be congruent. Draw diagrams to show that this is true.

2. *STUVWXYZ* is a cube. Write a paragraph proof that would convince someone that $\triangle STX$, $\triangle UTX$, and $\triangle STU$ are all congruent to one another.




## CUMULATIVE REVIEW CHAPTERS 1–4

### Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1. Which of the following is an illustration of the associative property of addition?

(1) $3(4 + 7) = 3(7 + 4)$
(2) $3(4 + 7) = 3(4) + 3(7)$
(3) $3 + (4 + 7) = 3 + (7 + 4)$
(4) $3 + (4 + 7) = (3 + 4) + 7$

**2.** If the sum of the measures of two angles is 90, the angles are
(1) supplementary.
(2) complementary.
(3) a linear pair.
(4) adjacent angles.

**3.** If $AB + BC = AC$, which of the following may be false?
(1) $B$ is the midpoint of $\overline{AC}$.
(2) $B$ is a point of $\overline{AC}$.
(3) $B$ is between $A$ and $C$.
(4) $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are opposite rays.

**4.** If $b$ is a real number, then $b$ has a multiplicative inverse only if
(1) $b = 1$ (2) $b = 0$ (3) $b \geq 0$ (4) $b \neq 0$

**5.** The contrapositive of "Two angles are congruent if they have the same measures" is
(1) Two angles are not congruent if they do not have the same measures.
(2) If two angles have the same measures, then they are congruent.
(3) If two angles are not congruent, then they do not have the same measures.
(4) If two angles do not have the same measures, then they are not congruent.

**6.** The statement "Today is Saturday and I am going to the movies" is true. Which of the following statements is false?
(1) Today is Saturday or I am not going to the movies.
(2) Today is not Saturday or I am not going to the movies.
(3) If today is not Saturday, then I am not going to the movies.
(4) If today is not Saturday, then I am going to the movies.

**7.** If $\triangle ABC \cong \triangle BCD$, then $\triangle ABC$ and $\triangle BCD$ must be
(1) obtuse. (2) scalene. (3) isosceles. (4) equilateral.

**8.** If $\overleftrightarrow{ABC}$ and $\overleftrightarrow{DBE}$ intersect at $B$, $\angle ABD$ and $\angle CBE$ are
(1) congruent vertical angles.
(2) supplementary vertical angles.
(3) congruent adjacent angles.
(4) supplementary adjacent angles.

**9.** $\angle LMN$ and $\angle NMP$ form a linear pair of angles. Which of the following statements is false?
(1) $\text{m}\angle LMN + \text{m}\angle NMP = 180$
(2) $\angle LMN$ and $\angle NMP$ are supplementary angles.
(3) $\overrightarrow{ML}$ and $\overrightarrow{MP}$ are opposite rays.
(4) $\overrightarrow{ML}$ and $\overrightarrow{MN}$ are opposite rays.

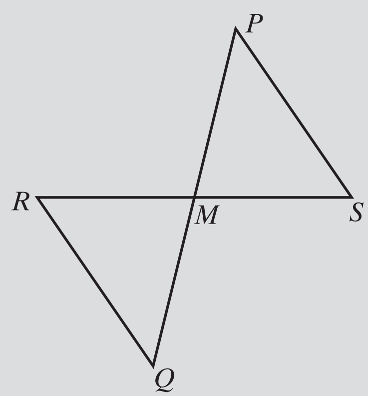
**10.** The solution set of the equation $3(x + 2) < 5x$ is
(1) $\{x \mid x < 3\}$
(2) $\{x \mid x > 3\}$
(3) $\{x \mid x < 1\}$
(4) $\{x \mid x > 1\}$

## Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

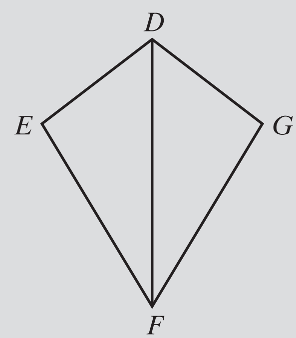
**11.** *Given*: $\overline{PQ}$ bisects $\overline{RS}$ at $M$ and $\angle R \cong \angle S$.

*Prove*: $\triangle RMQ \cong \triangle SMP$




**12.** *Given*: Quadrilateral $DEFG$ with $DE = DG$ and $EF = GF$.

*Prove*: $\triangle DEF \cong \triangle DGF$




## Part III

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

**13.** The following statements are true:

If our team does not win, we will not celebrate.

We will celebrate or we will practice.

We do not practice.

Did our team win? Justify your answer.

**14.** The two angles of a linear pair of angles are congruent. If the measure of one angle is represented by $2x - y$ and the measure of the other angle by $x + 4y$, find the values of $x$ and of $y$.

## Part IV

Answer all questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

**15.** Josie is making a pattern for quilt pieces. One pattern is a right triangle with two acute angles that are complementary. The measure of one of the acute angles is to be 12 degrees more than half the measure of the other acute angle. Find the measure of each angle of the triangle.

**16.** Triangle $DEF$ is equilateral and equiangular. The midpoint of $\overline{DE}$ is $M$, of $\overline{EF}$ is $N$, and of $\overline{FD}$ is $L$. Line segments $\overline{MN}$, $\overline{ML}$, and $\overline{NL}$ are drawn.

**a.** Name three congruent triangles.

**b.** Prove that the triangles named in **a** are congruent.

**c.** Prove that $\triangle NLM$ is equilateral.

**d.** Prove that $\triangle NLM$ is equiangular.